

## 1. Research Objective

This paper proposes a methodology to characterize uncertainty in automated vehicle (AV) dynamics in real time via Bayesian inference. Based on the estimated uncertainty, the method aims to continuously monitor the car-following performance of the AV to support strategic actions to maintain a desired performance. The sequential components in our methodology are:

- Stochastic Gradient Langevin Dynamics (SGLD) for real time estimation of vehicular dynamics uncertainty
- Dynamic monitoring of car-following stability
- Strategic action for control adjustment if anomaly is detected

## 2. Background

### I. General Longitudinal Vehicle Dynamics (GLVD)

Our estimation technique uses the GLVD general formulation below to capture two key uncertainty parameter relative to vehicle dynamics;  $T_L$  which is the actuation lag and  $K_L$  the ratio of the demanded acceleration that can be realized. Accordingly, the method would use real time sensor data on jerk ( $\dot{a}(t)$ ), acceleration  $a(t)$ , and controller demanded acceleration  $u(t)$ .

$$\dot{a}(t) = -\frac{1}{T_L}a(t) + \frac{K_L}{T_L}u(t)$$

## 3. Formulation

### I. The Bayesian Approach

Given a set of sensor data stream  $D_i$ , the goal is to estimate the posterior distribution in real-time of parameters  $T_L^t$  and  $K_L^t$ , defined as  $P(T_L^t, K_L^t | D_i)$ . This is extremely challenging in real-time setting given the non-linearity, unavailability of a closed formulation, and fast update frequency. Thus, typical probability sampling technique are ineffective.

We then resort to empirical methods for estimating  $P(T_L^t, K_L^t | D_i)$ . As such we use the Maximum a Posterior (MAP) formulation for our setting to achieve a the below stochastic formulation:

$$\min_{K_L^t, T_L^t} - \left( \log P(K_L^t, T_L^t) + \sum_i \log P(\dot{a}_i | a_i, u_i, K_L^t, T_L^t) \right)$$

Solving the above MAP provides a point estimate, yet what we require is estimating the entire posterior distribution, which means collecting multiple samples and then estimating the distribution empirically.

### II. The Stochastic Gradient Langevin Dynamics (SGLD)

To estimate the posterior distribution through the MAP, we develop an SGLD approach. The basic idea here is to iterate around the MAP solution, collect samples of the posterior distribution and then empirical estimate this posterior. Specifically, for our real-time estimation scheme at each iteration  $t$ , a mini-batch  $\{D^{(t,i)}\}_{i=1, \dots, N_t}$  of size  $N_t$  is used to update the SGLD parameters according to:

$$\nabla(K_L^t, T_L^t) = \frac{\eta_t}{2} \left( \nabla \log P(K_L^t, T_L^t) + \frac{N_t}{n} \sum_{i=1}^n \nabla \log P(\dot{a}_i | a_i, u_i, K_L^t, T_L^t) \right) + \varepsilon_t$$

$$\varepsilon_t \sim \mathcal{N}(0, \eta_t I)$$

#### Algorithm 1: SGLD Parameter Estimation

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1 Input:  $\theta^{(0)} = [K_L^0, T_L^0] \in R^2$ , initial step size  $\eta_1 > 0$ .
2 for  $k = 1, 2, \dots, K$  do
3   Randomly sample a subset of the minibatch  $\{D_{i=1, \dots, N_k}^{(k,i)}\}$  of size  $N_k$ ;
4   Compute the stochastic gradient  $\nabla(K_L^k, T_L^k)$ ;
5    $\eta_k \leftarrow \frac{\eta_1}{k}$ ;
6    $\theta^{(k)} \leftarrow \theta^{(k-1)} - \eta_k \nabla(K_L^k, T_L^k)$ ;
7 end for
8 for  $k > c$  do
9   Collect output  $\theta^{(k)} \sim P(K_L^t, T_L^t | D_i)$ 
10 end for

```

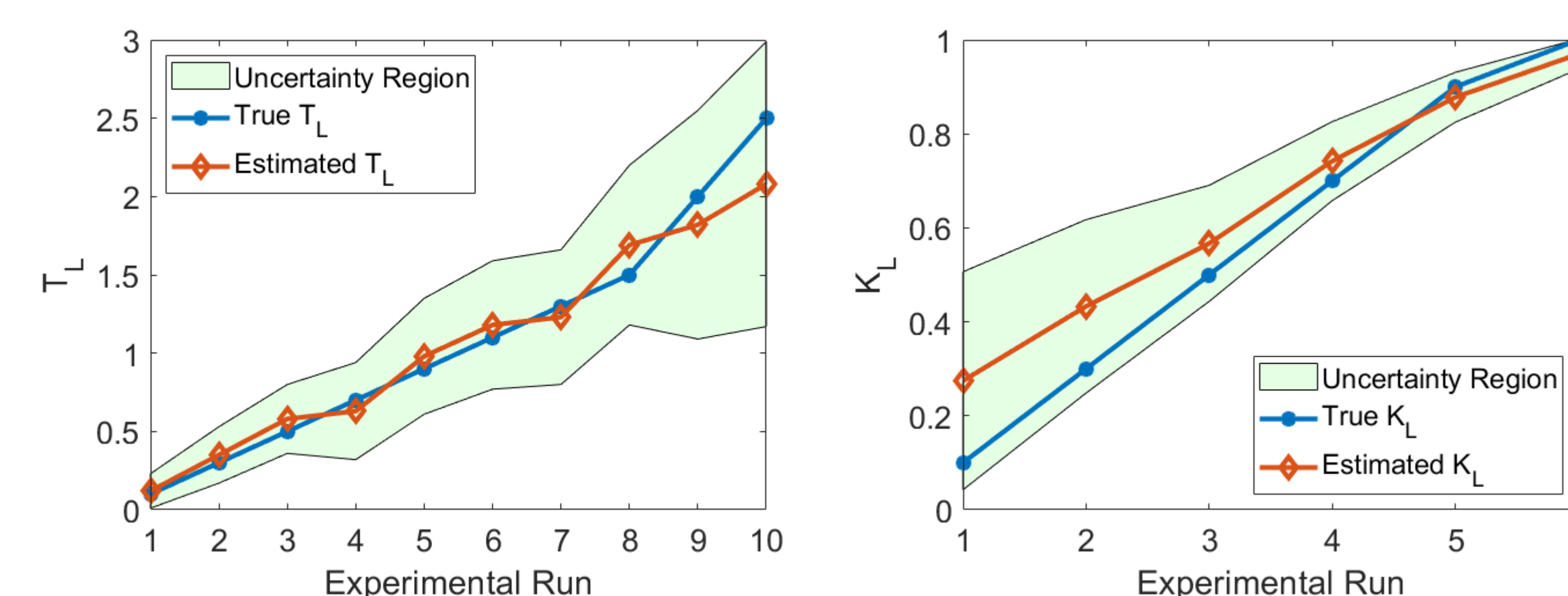
### III. Updating with New Data and Performance

Our approach is a dynamic and iterative one. Specifically, we are not just interested in estimating the posterior distribution  $P(T_L^t, K_L^t | D_i)$  but to continuously update it on the fly when new data is available. The idea is simple, we use the current posterior as a prior but with certain regularization. This regularization serves to penalize large deviations from the previous estimated posterior distribution. With the new prior we adjust our MAP formulation to fit the new setting.

$$\mathbb{P}_{prior}^{(t)} \sim \mathcal{N} \left( \left[ \bar{K}_L^{(t-1)}, \bar{T}_L^{(t-1)} \right]^T, \lambda I \right)$$

$$\min_{K_L^t, T_L^t} \left( \sum_{i=1}^{N_t} \left[ \dot{a}_i - \hat{a}_i \left( \left[ K_L^t, T_L^t \right] \right) \right]^2 - \frac{\sigma^2}{\lambda} \left\| \left[ \bar{K}_L^{(t)}, \bar{T}_L^{(t)} \right]^T - \left[ \bar{K}_L^{(t-1)}, \bar{T}_L^{(t-1)} \right]^T \right\|^2 \right)$$

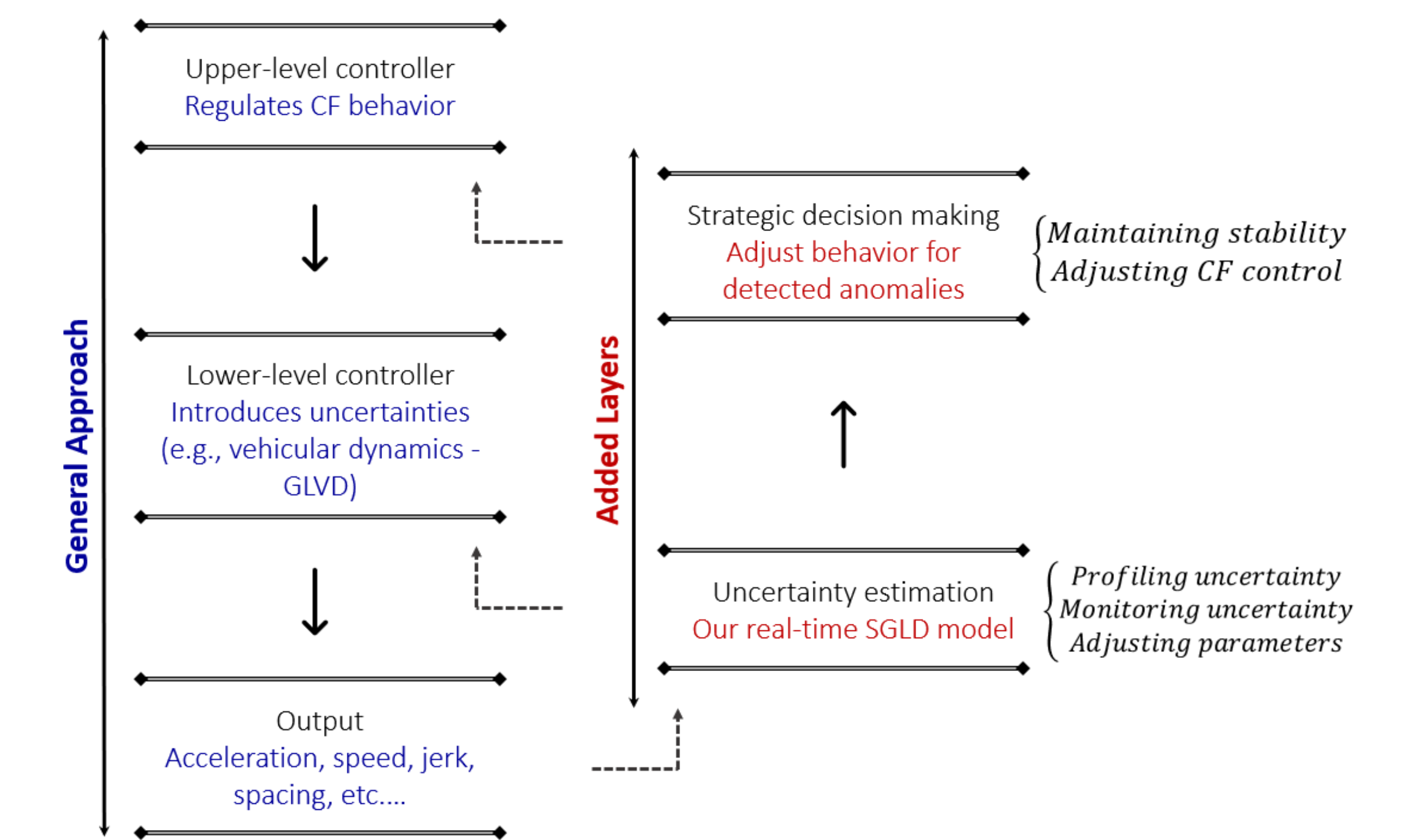
Thus, the output of our dynamic approach is an estimate of  $T_L$  and  $K_L$  along with uncertainty quantification around each prediction. The model performs well for both parameters. Error increases with large values of  $T_L$  and  $K_L$  which is expected as the controller is not performing well under there conditions.



## 4. Strategic Approach

### I. Strategic Approach and its Integration into the Control Design

The main idea here is to add a real-time data driven uncertainty estimation layer into the CF controller. This can be adopted for whatever control structure the vehicle is using. Such approach allows us to gauge the performance of the controller in real-time and adjust its behavior to recover desired performance.



### II. Online Algorithm

We build an online algorithm that encompasses the envisioned strategic approach. In here we focus to three main intervening strategies depending on severity of the issue: (i) adjust the parameters  $T_L$  and  $K_L$  inside the controller, (ii) update control gains, (iii) update time gap setting. Note that here we are showing a strategy for a typical linear controller, adjustments are needed for different control structure.

#### Algorithm 2: Online Strategy: Pseudo Code

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1 Input: Parameter estimates  $\theta^{(t)} = [K_L^{(t)}, T_L^{(t)}]$ 
2 if  $\theta^{(t)} \neq [K_L, T_L]$  then
3   Check stability (local and string)
4   if Stable then
5     Update lower-level controller:  $[K_L, T_L] \leftarrow [K_L^{(t)}, T_L^{(t)}]$ 
6   end if
7   if Unstable then
8     Update upper-level controller:
9     1. Update Control gains:  $[|k'_s| > |k_s|, |k'_v| > |k_v|, |k'_a| < |k_a|]^T \leftarrow [k_s, k_v, k_a]^T$ 
9     2. Update time gap:  $\tau^{*'} < \tau^* \leftarrow \tau^*$ 
9   end if
10 end if

```

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